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B.Sc-part-II, paper-III
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Differential Calculus

Q.1 State and prove the Euler's Theorem on partial differentiation of homogeneous function of two independent variables.

Soln: Statement: → If $f(x,y)$ be a homogeneous function of degree n, then

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

Proof: Let $f(x,y) = Ax^{\alpha_1}y^{\beta_1} + Bx^{\alpha_2}y^{\beta_2} + Cx^{\alpha_3}y^{\beta_3} + \cdots$ — (I)

Where, $\alpha_1+\beta_1 = \alpha_2+\beta_2 = \alpha_3+\beta_3 = \cdots = n$ — II

and $A, B, C, \cdots$ are constants i.e. independent of x and y.

Differentiating (I) partially with respect to x, we get

$$\frac{\partial f}{\partial x} = A\alpha_1 x^{\alpha_1-1}y^{\beta_1} + B\alpha_2 x^{\alpha_2-1}y^{\beta_2} + C\alpha_3 x^{\alpha_3-1}y^{\beta_3} + \cdots$$

$$\therefore x\frac{\partial f}{\partial x} = A\alpha_1 x^{\alpha_1}y^{\beta_1} + B\alpha_2 x^{\alpha_2}y^{\beta_2} + C\alpha_3 x^{\alpha_3}y^{\beta_3} + \cdots \quad \text{(III)}$$

Again, differentiating I partially w.r. to y, we get

$$\frac{\partial f}{\partial y} = A\beta_1 x^{\alpha_1}y^{\beta_1-1} + B\beta_2 x^{\alpha_2}y^{\beta_2-1} + C\beta_3 x^{\alpha_3}y^{\beta_3-1} + \cdots$$

$$\therefore y\frac{\partial f}{\partial y} = A\beta_1 x^{\alpha_1}y^{\beta_1} + B\beta_2 x^{\alpha_2}y^{\beta_2} + C\beta_3 x^{\alpha_3}y^{\beta_3} + \cdots \quad \text{(IV)}$$

Adding III and IV, we get

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = Ax^{\alpha_1}y^{\beta_1}(\alpha_1+\beta_1) + Bx^{\alpha_2}y^{\beta_2}(\alpha_2+\beta_2) + Cx^{\alpha_3}y^{\beta_3}(\alpha_3+\beta_3)$$

$$= Ax^{\alpha_1}y^{\beta_1}\cdot n + Bx^{\alpha_2}y^{\beta_2}\cdot n + Cx^{\alpha_3}y^{\beta_3}\cdot n + \cdots \quad \text{by II}$$

$$= n(Ax^{\alpha_1}y^{\beta_1} + Bx^{\alpha_2}y^{\beta_2} + Cx^{\alpha_3}y^{\beta_3} + \cdots)$$

$$= n f(x,y) \quad [\text{from (I)}]$$

Proved

B.Sc - part - II(H), paper - III
Differential Calculus

Q.2. If $x^3+y^3-x^3y^2z=0$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $x=y=1$.

Soln: We have, $x^3y^2z = x^3+y^3$

$$\therefore z = \frac{x^3+y^3}{x^3y^2} = \frac{1}{y^2}+\frac{y}{x^3}$$

$$\therefore \frac{\partial z}{\partial x} = -3\frac{y}{x^4}, \quad \frac{\partial z}{\partial y} = -\frac{2}{y^3}+\frac{1}{x^3}$$

$\therefore$ When $x=y=1$,

$$\frac{\partial z}{\partial x} = -3, \quad \frac{\partial z}{\partial y} = -2+1 = -1.$$

Q.3. If $u = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

Soln. We have, $u = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$

$$\Rightarrow u = x^2(y-z)+y^2(z-x)+z^2(x-y) \quad \text{——} I$$

Differentiating partially with respect to x, keeping y and z constants, we get

$$\frac{\partial u}{\partial x} = 2x(y-z)+y^2(-1)+z^2\cdot 1 = 2xy-2zx-y^2+z^2$$

Similarly, $\frac{\partial u}{\partial y} = x^2\cdot 1+2y(z-x)+z^2(-1) = x^2+2yz-2xy-z^2$

and $\frac{\partial u}{\partial z} = x^2(-1)+y^2\cdot 1+2z(x-y) = -x^2+y^2+2zx-2yz$

$$\therefore \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = \cancel{2xy}-\cancel{2zx}-\cancel{y^2}+\cancel{z^2}+\cancel{x^2}+\cancel{2yz}-\cancel{2xy}-\cancel{z^2}-\cancel{x^2}+\cancel{y^2}+\cancel{2zx}-\cancel{2yz}$$

$$\Rightarrow \frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$$

Proved.